DO NOW

Review your homework from Section 3.6 to see if there were any questions before we move on to the next section.

3.7 Related Rates

Related rates are used to find the rate of change of one value with respect to time, given the rate of change of a related value with respect to time.

An equation that associates the two values must be found. Implicit differentiation (the chain rule) should be used to differentiate each side of the equation with respect to time.

If the variable is *v*, then the derivative with respect to time, or rate of change of *v*, is *dv*/*dt*.

When working with related rates try to distinguish between the "general situation" and the "specific situation".

General situation – the property (equations) that are true at every instant or moment in time

Specific situation – the properties that are true at only a particular instant of time (Time is "frozen".)

Guidelines For Solving Related-Rate Problems:

1. Identify all *given* quantities and quantities *to be determined.* **MAKE A SKETCH** and label the quantities. Because rates involve variables and not constants, be careful not to label a quantity with a number unless it **NEVER** changes.
   ***This is the most important step!!!!

2. Find a formula or equation that relates the variables for the general situation. Eliminate unnecessary variables; these may be constants or eliminated because of a known relationship among the variables (using secondary equations).

3. Differentiate this equation implicitly with respect to time *t*.

4. Substitute the specific situation numerical values and solve for the required rate.

5. Label the answer with the appropriate units.

Example:

1. Assume that x and y are both differentiable functions of t and are related by the equation $y = 2(x^2 - 3x)$. Find $\frac{dy}{dt}$ when $x = 3$, given that $\frac{dx}{dt} = 2$.

$y = 2x^2 - 6x$

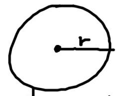
$\frac{dy}{dt} = 4x\frac{dx}{dt} - 6\frac{dx}{dt}$

$\frac{dy}{dt} = 4(3)(2) - 6(2)$

$\frac{dy}{dt} = 24 - 12$

$\frac{dy}{dt} = 12$

2. Petroleum from a leaking offshore oil well forms a circular slick on the ocean's surface. If the area of the oil slick is increasing at a rate of 5 km^2 per day, at what rate is the radius of the oil slick increasing at the instant the radius is 3 km.



Find $\frac{dr}{dt}$ when $r = 3$ km

$\frac{dA}{dt} = 5$ km^2/day

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$5 = 2\pi(3)\frac{dr}{dt}$

$\frac{5}{6\pi} = \frac{dr}{dt}$

$\boxed{\frac{5}{6\pi} \text{ km/day}}$

HOMEWORK

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Worksheet: Homework 3.7; 1 - 7